81. Proposed by CHAS. C. CROSS, Laytonsville, Md.

A circle is drawn bisecting the lines joining the points of contact of the escribed circles with the sides produced. Another circle is drawn passing through the centers of the circles drawn tangent externally to the in-circle and internally to the sides of the triangle. Prove that the centers of these two circles, the incenter and the circumcenter are collinear.

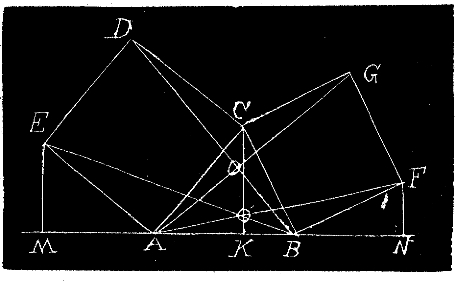
This problem is reprinted to correct an error in its enunciation. Inscribed is changed to *escribed*. EDITOR.

82. Proposed by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, Cal.

If the extremities of the base of a triangle be joined by straight lines to the exterior angles of squares constructed upon its two sides, the superior pair of lines thus drawn intersect at right angles; the inferior pair intersect at a point in a line drawn from the vertical angle perpendicular to the base.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.; COOPER D. SCHMITT, A. M., Professor of Mathematics in the University of Tennessee, Knoxville, Tenn.; and CHAS. C. CROSS, Laytonsville, Md.

1. In the two triangles BCD and ACG, we have $CG=CB$, $CD=CA$, $\angle BCD=\angle ACG$.



$\therefore \triangle BCD=\triangle ACG$ and $\angle BDC=\angle CAG$.

Hence, in quadrilateral $EAOD$, $\angle EAO+\angle EDO=$two right angles.

$\therefore \angle AED+\angle AOD=$two right angles, but $\angle AED=$a right angle and therefore AOD is a right angle.

$\therefore AG$ and BD are perpendicular to each other.

2. Let $AK=d$, $CK=f$. Then $BN=CK=f$, $FN=KB=b-d$, $AM=CK=f$, $EM=AK=d$.

$\therefore$ Equation to AF is $y=\dfrac{b-d}{b+f}x$, equation to BE is $y=-\dfrac{d}{b+f}(x-b)$.

These lines intersect at a distance $x=d$.

$\therefore AF$ and BE intersect on CK, the perpendicular from C on AB.

II. Solution by HENRY HEATON, M. Sc., Atlantic, Iowa; J. SCHEFFER, A. M., Hagerstown, Md.; and J. C. GREGG, Superintendent of Schools, Brazil, Ind.

Let $ACDE$ represent the square erected on the side AC, and $BCGF$ that on BC. Let O be the point of intersection of BD and AG, and O' that of BE and AF. In the triangles ACG and BCD, we have $CG=BC$, $AC=CD$, $\angle ACG=\angle BCD$. $\therefore \triangle ACG=\triangle BCD$. $\therefore \angle CDB=\angle CAG$, consequently the points A, O, C, D, are concyclic, and since $\angle DCA$ is a right angle, DOA must be a right angle.

Let EM and FN be the perpendicular let fall from E and F, respectively, upon AB produced. $O'H$ the perpendicular let fall from O' upon AB, and H' the foot of the perpendicular from C upon AB. We have

$$EM : O'H = MB : BH, \text{ and}$$
$$O'H : FN = AH : AN.$$
$$\therefore EM : FN = MB \times AH : AN \times BH,$$

but $MB = AM + AB = CH' + AB$, and $AN = BN + AB = CH' + AB$.

$\therefore MB = NA$, $\therefore EM : FN = AH : BH$, but $EM = AH'$, $NF = BH'$.

$\therefore AH' : BH' = AH : BH$.

$\therefore AH = AH'$, $BH = BH'$. Q. E. D.

CALCULUS.

63. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

What is the volume removed by boring an auger hole radius r through a right cylinder radius R, the center of the auger hole to pass at a distance c from the axis of the cylinder and inclined to the axis at an angle α?

I. Solution by the PROPOSER.

Let the axis of the cylinder whose radius is R coincide with the y-axis and let the axis of the cylinder whose radius is r intersect the z-axis at a distance c from the xy-plane, being parallel to the xy-plane and making an angle α with the y-axis. Pass a plane parallel to the xy-plane through the solid common to the two cylinders and at a distance z from the origin of coördinates. The intersection of this plane with the surface of the two cylinders forms a parallelogram, whose length is $2\sqrt{R^2-z^2}$ and whose width is $2\csc\alpha\sqrt{r^2-(z-c)^2}$. Hence its area is

$$\csc\alpha\sqrt{[r^2-(z-c)^2][R^2-z^2]}.$$

$\therefore V = \int_{c-r}^{c+r} \csc\alpha\sqrt{(R^2-z^2)[r^2-(z-c)^2]}dz$. Let $y = \dfrac{p+qy}{1+y}$, p and q to be determined from the conditions that the odd powers of z in the expansion under the radical shall vanish. From this condition we find $pq = R^2$ and

$$p+q = \frac{R^2-r^2+c^2}{c} \text{ or } \frac{R^2+r^2-c^2}{r}$$

according as $c > r$ or $< r$. From these two equations we find $p =$

$$\frac{R^2-r^2+c^2+\sqrt{(R^2-r^2+c^2)^2-4R^2c^2}}{2c} \text{ or } \frac{R^2+r^2-c^2+\sqrt{(R^2+r^2-c^2)^2-4R^2r^2}}{2r}$$

and q has the conjugate value of p. Making the substitution of $(p+qy)/(1+y)$, the values of p and q being replaced by their values in terms of R, r, and c as found above. The expression for the volume reduces to the following form :